Googol

Good morning folks,

In the first of these assemblies, I said the following: I'd like these assemblies to play a part in our responsibility to make you cleverer. By cleverer I mean know more stuff and be able to make links between the things you know and be able apply and use the things you know. Unlike your lessons these assemblies will not be constrained by a specification, which will give us the opportunity to get you thinking about a broader range of things than your lessons can. My dearest hope is we may mention things in these assemblies that you do not know, and those things may pique your curiosity such that you go and read about them for yourselves.

This assembly is very much written to do the above. Furthermore, I would like to try and highlight the wonder of numbers, human limitations when it comes to thinking about really big numbers and to consider whether we can comprehend the concept of infinity.

Elements of this assembly have been directly plagiarised from a television program I once saw presented by Carl Sagan. Carl Sagan, born in 1934, was a renowned American Astronomer and Science Communicator. In his Science Communicator role, he produced brilliant television programs and wrote some marvellous books. His great skill was to use fascinating scientific and mathematical discoveries to remind us of our humanity and our hitherto unique ability to unravel the secrets of the universe we live in. I said I wanted to pique your curiosity. Carl Sagan said "somewhere, something incredible is waiting to be known.

He was wonderful. You can still find his videos on YouTube and I strongly recommend you look them up.

Elements of this assembly are also based on my response to children who state, wrongly, that they do not like maths. Maths is fascinating and wonderful.

The story starts with the way we write numbers. I'd like you to try and picture the following in your head.

Ten. A one with a single zero after it. Easy to write, easy to picture. 10 apples, or 10 pound coins.

One hundred. A one with two zeros after it. Just two zeros, but one hundred likes on Instagram might feel like quite a few.

One thousand. A one with three zeros. The whole school out on fire drill is about one thousand. Still quite easy to picture in our minds.

One million. A one with six zeros. Can you picture one million of something? I am not sure I can. And this is just six zeros.

One billion. A thousand million. One with nine zeros after it. An eighth of the human population of planet Earth. A very, very big number. Only 9 zeros, but very difficult to picture in a meaningful way.

In the early part of the twentieth century a mathematician, Edward Kasner was chatting with his nine-year-old nephew Milton about very, very big numbers. They wondered about the number one with one hundred zeros after it. Kasner explained this number did not have a name. Perhaps Milton would like to name it. Milton suggested Googol. G O O G O L. The name stuck. So, a one with six zeros is a million, nine zeros is a billion, twelve zeros is a trillion, one hundred zeros is a Googol. A different spelling to the search engine Google, but Googol is clearly the source of the name Google.

It is quite easy to picture the number written down. One with a hundred zeros. It takes about thirty seconds to write it down if you do not worry too much about presentation. I know because I've tried it. But can we possibly picture a googol of something. One billion (9 zeros)

is an eighth the population of earth. Difficult to picture, but we do at least have a reference point. But a googol (one hundred zeros) of something? Is it possible, if we think really hard, for the human brain to comprehend a googol of something? I think it might not be, and I'll try to show you why not.

We could write a googol in standard form. We would write the number 10 and then the number 100 in superscript to the right of the 10. Much quicker than writing 100 zeros. What about the number that would be a one with a googol zeros after it. Not a hundred zeros, a googol zeros. It is called a googolplex. This too would be easy in standard form. We could write a 10, then a 10 in superscript to the right of the original 10, then 100 in super superscript next to the superscript 10. One with a googol zeros after it. Do you remember I said writing a googol in full, one hundred zeros, is fairly simple. Thirty seconds work. Well, if you were try and write out a googolplex in full. A one with a googol zeros after it, you would find there is not enough space in the entire universe to write it down. Even if each zero were the size of an atom. And it

would take far longer than the age of the universe. A googol is unimaginably huge. And a googolplex many, many times huger.

I love the expression mind boggling. The feeling I get when I try to think about a googol, and about googolplex, is very much mind boggled. Awe, incomprehension, a feeling that no matter how hard I think a full understanding will always elude me.

But wait. There is more. How ever mind bogglingly large the number googolplex is, it remains as good as zero when compared to infinity. Something boundless, with no end, bigger than any real number may be described as infinite. Infinitely bigger than googolplex.

As all children do, I used to think of infinity as the biggest possible, the ultimate mind boggling number. Until I read about a thought experiment proposed by a German mathematician, David Hilbert in 1924. He wanted to demonstrate there are different types, and even different sizes, of infinity. It goes like this:

Imagine a hotel with an infinite number of rooms. One evening a bus pulls up outside. The bus contains an infinite number of guests. The manager rubs her hands with glee. It is going to be a tremendously profitable night. Every one of the infinite guests is allocated a room and the hotel is full. The hotel is full.

A little later, a new potential guest arrives. The hotel manager is never one to turn away business. She is also an excellent mathematician. I imagine her rather like Ms Stanger. She realises the hotel can still accommodate this guest. She simply asks every one of the guests to move to the room number which is one higher than their own. Room number 1 to room number 2. Room 2 to room three. Room n, to room n + 1. The new guest gets room 1, and all infinite other guests still have a room. Now the hotel is full.

The next evening the hotel manager is informed all infinite guests wish to stay a second night. Just as she finishes taking payment, another bus

pulls up outside with an infinite number of guests. Well now, this is a trickier problem. But still far too good an opportunity to turn down. Our super smart protagonist puts her mind to it. And before long turns up an excellent solution. The manager asks each current guest to take their room number and double it. Then move to the resulting room number. Two goes to four, three to six, seven to fourteen and so on, infinitely. The result of this mathematical magic is every one of the original guests is now in an even numbered room. An infinite number of even numbers. Leaving an infinite number of odd numbered rooms for the infinite guests waiting patiently outside. Infinitely more profitable than the previous infinitely profitable evening.

I am rather fond of this paradox. I offer it to you as another example of something that is mind boggling. Though, despite the presence of infinity in the paradox, the simple, familiar nature of the maths, n+1, or 2n giving us all even answers, gives our brains something to cling to, which googol and googolplex do not offer.

I hope this assembly so far has achieved my stated aims of highlighting the wonder of numbers, and human limitations when it comes to thinking about really big numbers. And to consider whether we can comprehend the concept of infinity.

Though, as I was writing this assembly two other themes arose, one coincidentally and the other as I did a bit of research on Carl Sagan making sure I get my facts and quotes right.

The coincidental one: I was at a meeting of headteachers and two researchers into dementia were delivering a talk. They shared some of the wonderful voluntary work they do, which involved bringing small groups of people with dementia into schools to work with pupils. They spoke passionately about the benefits of this program for the people with dementia, the pupils and the community. And they were keen to share strategies for warding off the onset of dementia. One of which was to use your brain or lose it. The more time you spend thinking hard

and being mind-boggled the healthier your brain will be. I thought this an appropriate assembly to share this advice.

The Carl Sagan research theme was based on the following quote:

"What an astonishing thing a book is. It's a flat object made from a tree with flexible parts on which are imprinted lots of funny dark squiggles. But one glance at it and you're inside the mind of another person, maybe somebody dead for thousands of years. Across the millennia, an author is speaking clearly and silently inside your head, directly to you. Writing is perhaps the greatest of human inventions, binding together people who never knew each other, citizens of distant epochs. Books break the shackles of time. A book is proof that humans are capable of working magic."

Always read. Reading is the best way to get cleverer and to stay clever. Reading is the best way to find those incredible things waiting to be known. The best way to find those things that will amuse, entertain and

boggle your mind. The best way to make sure your brains stays healthy.

I commend Carl Sagan to you, and the number googol.

Thank you for listening.